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# B.M.S. COLLEGE FOR WOMEN, AUTONOMOUS

**BENGALURU – 560004**

**SEMESTER END EXAMINATION – SEPT/OCT 2023**

**M.Sc in Mathematics – 4th Semester**

## MAGNETOHYDRODYNAMICS

**Course Code: MM407T** **QP Code: 14007**
**Duration: 3 Hours** **Max marks: 70**

***Instructions: 1) All questions carry equal marks.***
***2) Answer any FIVE full questions.***

1. (a) State and explain Faraday's law of induction and show that $\nabla \times E = \frac{-\partial \vec{B}}{\partial t}$, where the quantities have their usual meaning.

   (b) Derive Gauss law for dielectric materials in its standard form. **(7+7)**

2. (a) Prove that the tangential component of the electric field is continuous across the Interface.

   (b) Using Ampere's law deduce Vector potential, Scalar potential and Lorentz force. **(7+7)**

3. (a) Derive the equation of conservation of momentum in its usual form

   (b) Derive the magnetic induction equation in the form $\frac{\partial \vec{B}}{\partial t} = \nabla \times \left(\vec{q} \times \vec{B}\right) + \frac{1}{R_m}\nabla^2 \vec{B}$, where the quantities have their usual meaning. **(7+7)**

4. (a) Explain the concept of frozen-in-phenomenon. Establish this fact by providing the appropriate theorem.

   (b) Show that the angular velocity of a perfectly conducting fluid body rotating steadily about the axis of symmetry in a magnetic field does not change along the magnetic field lines. **(7+7)**

5. (a) Define force free magnetic field. Then prove the following.

   (i) $\alpha = \frac{\vec{B}.(\nabla\times\vec{B})}{|\vec{B}|^2}$

   (ii) $\alpha = \frac{\vec{B}.(\nabla\times\vec{B}).\nabla\times(\nabla\times\vec{B})}{|\nabla\times\vec{B}|^2}$

(b) State and prove Chandrashekar's theorem on force free magnetic field. **(6+8)**

6. (a) In an infinitely conducting fluid moves parallel to the vertical z-axis of the Cartesian co-ordinate system $(x, y, z)$ with velocity $w$ and the magnetic field lines act in y-direction and all the variables are independent of $x$ and $y$ the show that $H$ and $\rho$ satisfies the following equations

$$\frac{Dh}{Dt} = -H\frac{\partial w}{\partial z} \quad and \quad \frac{D\rho}{Dt} = -\rho\frac{\partial w}{\partial z}.$$

(b) Explain Bennet pinch and any two instabilities associated with it. **(7+7)**

7. (a) Derive an Alfven wave equation in an incompressible perfectly conducting fluid in the presence of a suitable magnetic field.

(b) Derive equations of equi-partition of energy by Alfven's waves. **(7+7)**

8. Obtain the velocity and temperature distributions for one-dimensional Hartmann flow. Discuss the effect of magnetic field on the same. **(14)**

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